The value of the coefficient C in formula (2) for the measured values of q_0 and p_0' was chosen on the basis of calibration experiments with fluoroplastic-4 samples and was 0.15 (for argon).

The obtained distributions of pressures and heat fluxes over the cross section of the jet shown in Fig. 2a, b permit us to determine the zones of flow where the variations of q_0 and p_0' do not exceed 20%. This zone has a diameter of 14 mm; therefore, for conducting ablation investigations samples were prepared with diameter not exceeding this size. The ablation tests of the samples in the chosen operating regimes permitted us to obtain the dependence of the effective ablation heat H_{eff} on the deceleration enthalpy of the flow i_0 which, as is well known [9], determines the behavior of certain types of ablation materials.

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A THERMAL-DIFFUSION MODEL OF THE EROSION OF ELECTRODES IN AN MHD GENERATOR

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UDC 537.523: 538.4

We propose a diffusion-heat-conduction model of erosion at a cathode spot, on the basis of which we estimate the characteristic lifetimes of cathode spots and the erosion of electrodes made of graphite, copper, iron, and chromium under the conditions of an MHD generator. We show that the rate of erosion depends substantially on the geometric shape of the cavity.

The most complete measurements of the macroscopic parameters of cathode spots on copper electrodes in a stream of plasma with an easily ionizable additive were apparently made in [1]. The measurements indicated a heat flux of $Q_0 \simeq 20$ W/A for both cold and hot electrodes and a spot lifetime of $\tau \simeq (10^{-3}-10^{-2})$ sec, depending on the average current I = (2-5) A at the spot, where τ denotes the time required for the spot to move a distance of the order of its diameter; an erosion value of G = (0.1-0.7) μg/C and other parameters were also measured.

It seems unexpected that it is possible to obtain a $\tau(I)$ that agrees with the experimental results, starting only with the balance of electrode vapor concentrations in the region of the spot, if we take as the initial parameter the heat flux Q_0, which it is natural to correlate with the cathode potential drop, amounting to about 15-20 V, and therefore consider practically independent of time, electrode temperature, and other conditions, and

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 3, pp. 437-442, September, 1976. Original article submitted July 23, 1975.

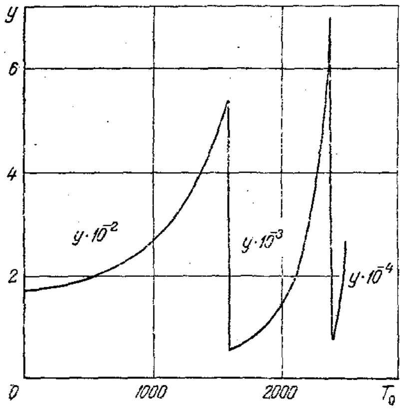

Fig. 1. Rate of erosion g^* (g/cm$^2\cdot$ sec) for chromium electrodes at various electrode temperatures T_0 (°K) at an average current density to the electrode of $j = 4.5$ A/cm^2 (β is a geometric factor equal to unity for a hemispherical cavity; $\nu \simeq 1.2$); $y = \gamma g^* \beta^{-4}$, μg/C.

also assume that the ionization of the electrode vapors plays a substantial role in the processes taking place at the spot.

Since we have no information concerning the distribution of heat flux over the surface of the cavity, we assume that the surface is hemispherical. An elementary solution of the stationary heat-conduction equation for cold electrodes with a given point source $Q = Q_0 I$ gives us the relation between temperature and radius:

$$T = \frac{Q}{2\pi \varkappa r}, \tag{1}$$

where $\varkappa$ is the average thermal conductivity.

The initial process of vaporization to R_b, corresponding to a boiling temperature T_b, at a pressure equal to external pressure, with gasdynamic emission of electrode vapors, takes much less time than τ and will not be considered here (the gasdynamic times τ^* are much shorter than the time required for establishing the thermal and diffusion processes, about 10^{-6}-10^{-5} sec, which is, in turn, much less than τ); the total pressure of the gas in the cavity is therefore constant and equal to the external pressure, about 1 atm. The partial pressure, p_{Cu}, of the vapors of a copper electrode near the surface of the cavity is determined by the vaporization and the diffusion of the vapors to the external boundary of the cavity, on which, as can be concluded from a comparison of the average rate of diffusion with the velocity of the plasma flux in a laminar boundary layer at a distance $\sim R_b$ from the surface of the electrode under the conditions of [1] and the average diffusion rate, p_{Cu} is close to zero.† The energy required for vaporization constitutes a negligibly small fraction of the heat flux. We assume that the cathode spot is fixed.

The balance of concentrations $N(r, \theta)$ is determined by the diffusion equation $\Delta N = 0$, which in spherical coordinates takes the form

$$\frac{\partial N}{\partial r}\left(r^2 \frac{\partial N}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial N}{\partial \theta}\right) = 0 \tag{2}$$

with the boundary conditions

$$N(r, \theta) = 0 \quad \text{for} \quad \theta = \pi/2; \tag{3}$$

$$\frac{A \exp\left(-\frac{B}{T}\right)}{\sqrt{2\pi k T m}} - N\sqrt{\frac{kT}{2\pi m}} = D\frac{\partial N}{\partial r} \quad \text{for} \quad r = R, \tag{4}$$

where D is the average coefficient of diffusion of the vapors at the cathode spot; $0 < \theta < \pi/2$, and the angle $\theta = \pi/2$ corresponds to the outer boundary of the cavity. The left-hand terms in (4), with A, B = const, describe

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† Taking account of the presence of convective flows even when there is no axial flow of gas, we can hope that the proposed model for erosion at the cathode spot is more general in nature and is applicable not only to the specific conditions of MHD generators.

the rate of vaporization [2] and the very substantial rate of reverse deposition of electrode-material vapors corresponding to it by the principle of detailed equilibrium. Since the available information is insufficient, in (4) we average with respect to θ, so that we can considerably weaken the requirements imposed on the solution of (2) and greatly simplify the problem.

Equations (2)-(4) are satisfied, in particular, by the two-parameter characteristic function of (2):

$$N(r,\theta)=Cr^nP_n(\cos\theta);\; n=1,\; 3,\; \ldots, \tag{5}$$

where the P_n are Legendre polynomials and C = const; the solutions with $n > 1$ may be discarded, since for these $N(r, \theta)$ may become negative, which is impossible by the physical nature of the problem, and in that case†

$$N(r,\theta)=\frac{\pi}{2}\,\frac{A\exp\left(-\dfrac{B}{T}\right)r\cos\theta}{kTR}\,. \tag{6}$$

The variation of the cavity radius with time is determined from the differential equation

$$d\left(\frac{2}{3}\,\pi R^3\rho\right)=-mD\frac{\overline{\partial N}}{\partial\vec{n}}\,S\,dt, \tag{7}$$

where $\vec{n}$ is the external normal to the projection of the cavity onto the electrode surface; $S = \pi R^2$; and the bar indicates averaging with respect to r. Making use of (1) and (6), we reduce (7) to the form

$$\frac{dR}{dt}=\frac{\pi}{4}\,\frac{mAD}{\rho kTR}\exp\left[-\frac{B}{T(R)}\right] \tag{8}$$

with the solution

$$R=\frac{\alpha}{B}\ln\left(\frac{\pi}{4}\,\frac{mABD}{\rho k\alpha^2}\,t+\exp\left(\frac{B}{\alpha}\,R_{\text{b}}\right)\right);\; \alpha\equiv\frac{Q}{2\pi\varkappa};\; R>R_{\text{b}}\,. \tag{9}$$

According to (1), (6), and (9), we find $N(r, \theta)$ as a function of time:

$$N(r,\theta,t)\simeq\frac{\pi^2}{8}\,\frac{\alpha\rho}{mBD}\,\frac{r\cos\theta}{t+\dfrac{4\rho k\alpha^2\exp\left(\dfrac{B}{T_{\text{b}}}\right)}{\pi mABD}}\,. \tag{10}$$

The quantity

$$\tau=\frac{8\rho k\alpha^2\exp\left(\dfrac{B}{T_{\text{b}}}\right)}{\pi mABD}=\frac{2\rho kQ^2}{\pi^3\varkappa^2 mBDp_0}\quad(p_0=1\ \text{atm}) \tag{11}$$

can be taken as the characteristic lifetime of the cathode spot; it corresponds to a reduction in the pressure of the saturating vapors by a factor of 3 and is found to be proportional to the square of the current at the spot. This time may be called the maximum time, since fluctuations of various kinds and processes not taken into account in the idealized case considered here may lead to a shortening of the lifetime of the spot. ‡ In the

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† We can convince ourselves without difficulty that since the diffusion rate is low, condition (4) can be replaced with

$$\frac{A\exp\left(-\dfrac{B}{T}\right)}{\sqrt{2\pi kTm}}-N\sqrt{\frac{kT}{2\pi m}}\simeq 0\quad\text{for } r=R,$$

which gives us a solution $N(r, \theta)$ differing from the alternative solution (6) in the fact that $N(R, \theta)$ depends on θ. As has been noted, the final choice between the solution is made more difficult by the lack of information concerning the distribution of heat fluxes over the surface of the cavity [among other things, we do not know the actual function $T(R, \theta)$ etc.].

‡ Both the lifetime and the erosion may be considerably reduced as a result of a transition to autoemission regime (the effect of the additive, surface films of oxides, etc.).

case of a copper electrode, if we take the gas-kinetic radius of a copper atom as ~ 1.3 Å, the gas-kinetic radii of the molecules of the medium will be ~ 1.5 Å [3]; $T_b = 2850$°K; $B = 39.3 \cdot 10^3$°K [2]; the average thermal conductivity, including the solid and liquid phases, will be $\varkappa \cong 2.5$ W/cm · deg [4, 7]; we find $D \simeq 3.5$ cm^2/sec, and in the 2-5 A current range $\tau = (1.2\text{-}8) \cdot 10^{-3}$ sec; $g \simeq (1.3\text{-}20) \cdot 10^{-2}$ μg.

Using an expansion in a series in powers of T_0/T, we could describe (1) in a more precise form as

$$T(r) \simeq T_0 + \frac{Q}{2\pi\varkappa r}\,. \tag{1a}$$

This makes it necessary to multiply the right side of (11) by the coefficient $(1 + T_0/T_b)^3$ ($T_0 = 400$°K).

Even though R changes little, the average rate of erosion of a copper electrode is determined by τ and, according to (1), (11), is equal to

$$G = \frac{g}{\tau I} = \frac{\pi Q_0 m T_b^3 \; B D p_0}{12\varkappa k (T_b^2 \; - T_0^2)^3} \simeq 5 \;\; \mu\text{g/C}, \tag{12}$$

where $p_0 = 1$ atm.

If we reduce the thermal conductivity, other conditions remaining the same, we find a proportional increase in the erosion rate G, which may take place when we change from copper to stainless steel. Thus, if we used the value $B = 46.2 \cdot 10^3$°K and $T_b = 3130$°K for iron [2], taking $\varkappa \simeq 0.3$ W/cm · deg [3], we would obtain $G = 32$ μg/C.

A more convenient quantity than G may be the average rate of erosion per unit of electrode surface

$$g^* \doteq \frac{G n I_0}{S_{el}} = \frac{\pi U_c j p_0 m B D}{12\varkappa k (T_b \; - T_0)^3 \left(1 + \dfrac{T_0}{T_b}\right)^3}\,, \tag{13}$$

where $n = I/I_0$ is the number of simultaneously existing cathode spots; I is the total current to the electrode; I_0 is the average current at an individual spot; and $U_c = 20$ V is the cathode drop corresponding to a heat flux Q_0.

Thus, under typical conditions of an MHD generator in the case of graphite electrodes with $j \cong 3$ A/cm^2, $T_0 = 1300$°K, $T_b = 4200$°K, $B = 92.3 \cdot 10^3$°K, $\varkappa \simeq 0.25$ W/cm · deg, and $D \simeq 18$ cm^2/sec we would obtain $g^* = 1.4 \cdot 10^{-4}$ g/cm^2 · sec.

Because of the possible practical use of chromium electrodes in MHD generators, the results of calculations of the rate of erosion of chromium are shown in Fig. 1 as a function of electrode temperature ($T_b = 2850$°K; $B = 46.3 \cdot 10^3$°K; $\varkappa = 0.3$ W/cm · deg; $D = 3.5$ cm^2/sec [2, 7]).

The temperature on the surface of the cathode spot is too low for the autothermoelectronic emission current to make any notable contribution to the current at the spot, $i \simeq (10^5\text{-}10^6)$ A/cm^2.

We can convince ourselves without difficulty that the ion current can be equal to i only for almost complete ionization of the vapors. Therefore, immediately at the surface there must exist a thin (possibly measuring some tens of mean free paths) nonequilibrium region of space charge and ionization, for which we have a cathode potential drop with concentrations of ions held by the field that are substantially greater than at external pressure $p_0 \sim 1$ atm. Even though the thickness l of this region is small, the high pressure inevitably means that there will be a corresponding rise in the boiling temperature at the surface, T_b^*, if a corresponding p_0 is maintained, $T_b < T_b^*$, on the boundary of l.

From the universality of the thermodynamic relation

$$N \propto \exp\left(-\frac{q}{kT}\right),$$

where $q \simeq B$ is the heat of sublimation, and from the fact that if there is no gasdynamic leakage, the energy required for the transition of an atom from the nonequilibrium region to the equilibrium region, with a field intensity of $E \sim 0$ in the latter, will again, in the final analysis, be equal to q, it follows that the boundary conditions in the diffusion equations (2)-(4) will be valid, where these are now satisfied not immediately on the surface of the spot, but on the boundary of l.

Since there is a rapid escape of vapors in the initial stages of the existence of the spot, the process whereby the electric-field forces balance the high pressure apparently existing at distances less than l near the surface, with subsequent cessation of the escape, must take a very small fraction of the total lifetime of the spot, and this justifies the proposed diffusion model.

It follows from (1) that the difference between T_b and T_b^* is due to the radial nonuniformity resulting from the nonequilibrium region l and may be taken into account by making the substitution

$$\varkappa \to \varkappa_{\text{eff}} = \varkappa T_b^* / T_b \equiv \varkappa\gamma$$

in all the formulas. Thus, when the pressure at the surface is 10 times p_0, we find $\gamma \simeq 1.2$ for copper. In spite of some decrease, $\tau \propto 1/\gamma^2$ does not go beyond the limits of accuracy of the experiment, and its effect on the erosion $g^* \propto 1/\gamma$ is even weaker.

The erosion values G calculated above, for example, for copper, exceed the experimental values [1, 5, 6] by one order of magnitude. One of the most probable reasons for this is that the geometric shape of the cavity is much flatter than hemispherical. Assuming a cavity depth $\sim r_b$ and a diameter $2R_b$, from the heat-conduction equation

$$\varkappa \frac{\partial T}{\partial r} \sim \varkappa \frac{T_b}{r_b} \sim \frac{Q}{R_b^2},$$

we find, calculating the volume of the cavity, that $V \propto r_b R_b^2$, and if we introduce the geometric factor $\beta \equiv r_b/R_b$ $V \propto \beta^1$. At the same time, τ is independent of β. Thus, we can obtain satisfactory agreement with experimental data if we reduce the depth of the cavity by a factor of about 2 (assuming that the geometric shape does not vary with time and taking account of γ).

Much of the work in this study was done in collaboration with G. V. Gofman, to whom I wish to express my sincere gratitude.

NOTATION

τ, lifetime of cathode spot; Q_0, heat flux to cathode spot (total thermal power per unit of current intensity); Q, heat flux to cathode spot (total thermal power); I, current intensity (in the arc or on the whole electrode); I_0, current intensity of arc; G, value of erosion (in terms of mass) at the cathode spot, per unit of electric charge passing through; T, temperature of vaporizing surface of cavity; T_0, initial temperature of electrode; R_b, radius of cavity R at which the temperature of its surface is equal to the boiling temperature of the electrode material; r, θ, spherical coordinates of points in the volume of the cavity; $\varkappa$, average thermal conductivity of the electrode material; N(r, θ), concentration of atoms of electrode vapors in the volume of the erosion cavity; A, B, constants in the formula giving the pressure of saturated vapors as a function of temperature; k, Boltzmann constant; ρ, density of electrode material; $P_n(\cos\theta)$, Legendre polynomial of order n in $\cos\theta$; m, mass of an atom of the electrode material; D, diffusion coefficient; t, time; T_b, boiling temperature of the electrode material at pressure equal to the pressure of the external medium (~1 atm); g, mass of eroding material per cavity; p_{Cu}, partial pressure of copper vapors; p_0, external pressure (~1 atm); S_{el}, electrode area; j, electrical current density averaged over the electrode; U_c, cathode potential drop; n, order of the Legendre polynomial, number of cathode spots simultaneously existing on the electrode; g*, rate of erosion (in terms of mass) per unit of electrode surface at a given mean current density over the electrode; l, thickness of nonequilibrium region of ionization near the surface of the cathode spot; β, geometric factor (ratio of twice the depth of the cavity to its diameter); T_b^*, boiling temperature at elevated cavity surface pressure resulting from the presence of the electric field; γ, correction factor, equal to $T_b^*/T_b \simeq 1.2\text{–}1.4$; V, volume of cavity; N, concentration of atoms in saturated vapors; r_b, effective depth of cavity.

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THERMAL-SHOCK PERFORMANCE OF EVAPORATED Al_2O_3, ZrO_2, AND LAMINATED CERMET COATINGS IN AN ELECTRIC-ARC PLASMA SOURCE

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UDC 533.6.071.1:62-69.
629.78.023.222

Measurements have been made on the thermal stability and performance of protective coatings deposited by plasma-gun techniques on copper with an arc heater.

The working materials in an arc heater are subject to very severe stress, the main causes of failure being thermal shock, erosion, surface reactions, and embrittlement [1]. Protective coatings can provide reliable operation at high temperatures, since they restrict oxidation, improve the stability under erosion, and provide control over the heat transfer to the circulating coolant by conduction.

The performance of such a coating is dependent on the rates of heat transfer and the nature of the substrate. At present, we have no coatings that can be used at all the temperatures that occur in such heaters, but such coatings are of value even at moderate temperatures, since they ease the strain on the working materials, optimize the cooling conditions, and reduce the heat loss.

The coatings most widely used are Al_2O_3 and ZrO_2; these are stable in oxidizing atmospheres and have high melting points (~ 2300°K), while their thermal conductivities are relatively low, $0.5 < \lambda < 4$, and industrial techniques are available for deposition [2].

The working gas parameters in such heaters are $1000 < T_0 < 5000$°K and $P_0 > 50 \cdot 10^5$ N/m^2, and the heat loss through the discharge chamber increases with the pressure, and so the components have to be protected from high heat fluxes; also, the continuous working time is such that a steady-state temperature distribution is usually set up.

This steady state is reached in a short time, and therefore a thermal shock occurs on switching the device on; naturally, the coating material must be resistant to such shocks.

The deposition technique must ensure good adhesion to the substrate; if the bond is poor, the coating may fail for mechanical or thermal reasons, and also on account of differences in linear expansion coefficients.

TABLE 1. Deposition Conditions

Material	Nozzle diameter, mm	Gas flow, liters/sec		Gas used to supply powder	Current, A	Voltage, V	Distance from end of nozzle to workpiece, mm
		Ar	H_2				
Al_2O_3	5—7	30—60	6—8	3—6	350	60	100—130
ZrO_2		40—50	4—5		460	60—75	100—120
NA-67		30—50	6—8		350	60	200—250

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 3, pp. 443-448, September, 1976. Original article submitted July 23, 1975.